Quiz 10

February 24, 2016

Evaluate the following integrals. Write "diverges" if the integral diverges.

(a) $\int_1^\infty \frac{12}{\sqrt[3]{x^4}}dx = \lim_{a\to\infty} \int_1^a 12x^{-4/3}dx$

$= \lim_{a\to\infty} \left[12\left(\frac{x^{-1/3}}{-1/3}\right)\right]_1^a$

$= \lim_{a\to\infty} \left[-\frac{36}{\sqrt[3]{x}}\right]_1^a$

$= \lim_{a\to\infty} -\frac{36}{\sqrt[3]{a}} + \frac{36}{\sqrt[3]{1}} = 0 + 36 = \boxed{36}$

(b) $\int_0^\infty \frac{3x}{e^{x^2}}dx = \lim_{a\to\infty} \int_0^a 3xe^{-x^2}dx$

$u = -x^2$
$du = -2x\,dx$

$= \lim_{a\to\infty} \int_{x=0}^{x=a} 3xe^u \cdot \frac{du}{-2x}$

$= \lim_{a\to\infty} -\frac{3}{2}\int_{x=0}^{x=a} e^u du$

$= \lim_{a\to\infty} -\frac{3}{2}e^u \Big|_{x=0}^{x=a}$

$= \lim_{a\to\infty} -\frac{3}{2}e^{-x^2}\Big|_{x=0}^{x=a}$

$= \lim_{a\to\infty} -\frac{3}{2}e^{-a^2} + \frac{3}{2}e^0 = 0 + \frac{3}{2} = \boxed{\frac{3}{2}}$